

Q1 (10 points). For the control system of figure (1-a):

1. If the control strategy is a proportional control with an amplification term  $K$ .
  - a. Find the overall transfer function of the system.
  - b. Find the open-loop transfer function of the system.
  - c. The characteristic equation of the system.
  - d. Find the values of  $K$  that keeps stable this control system.
  - e. Find the values of  $K$  that keeps the roots of the characteristic equation of this system out of the relative stability zone (refer to figure (1-b)).

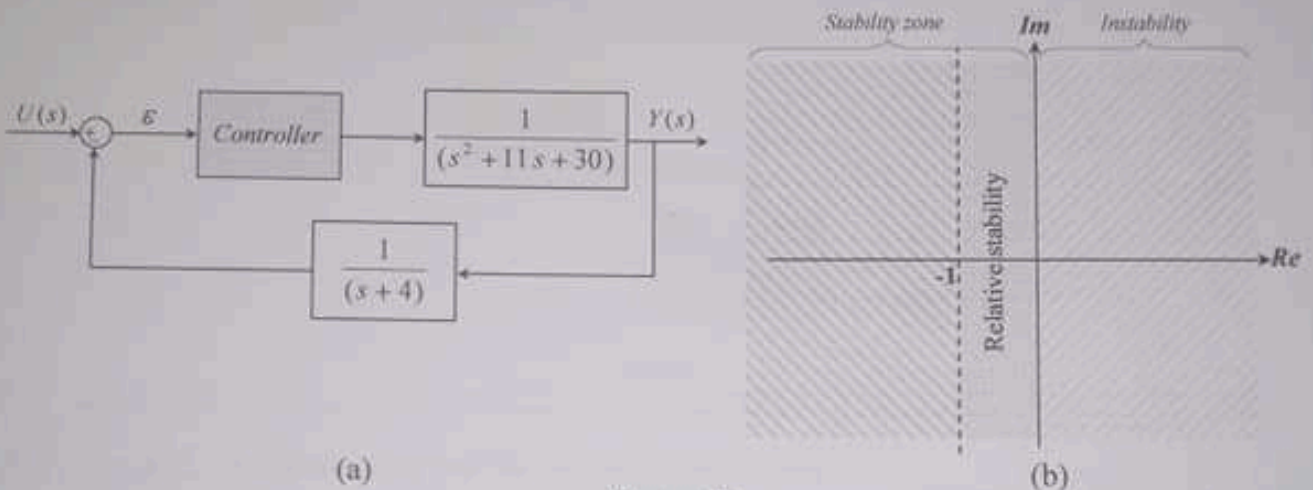


Figure (1)

2. If we want to use the PID controller schematized in figure (2). Using the Ziegler-Nicholas methods, what values of  $K_C$ ,  $T_I$  and  $T_D$  we have to use to obtain an optimal response of the system.

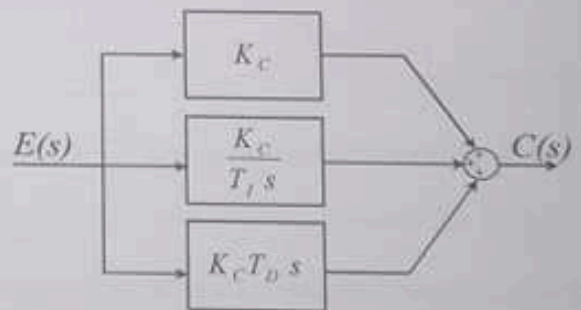


Figure (2)

Q2 (10 points). For the system described by the block diagram of figure (3), with  $G_1(s) = 3$ ,  $G_2(s) = \frac{1}{(s+2)}$ ,  $G_3(s) = (s+1)$ ,  $G_4(s) = \frac{1}{(s+1)}$ ,  $H_1(s) = \frac{1}{(s+3)}$ ,  $H_2(s) = \frac{1}{(s+1)}$  and  $H_3(s) = 1$ . Find the overall transfer function of the system.

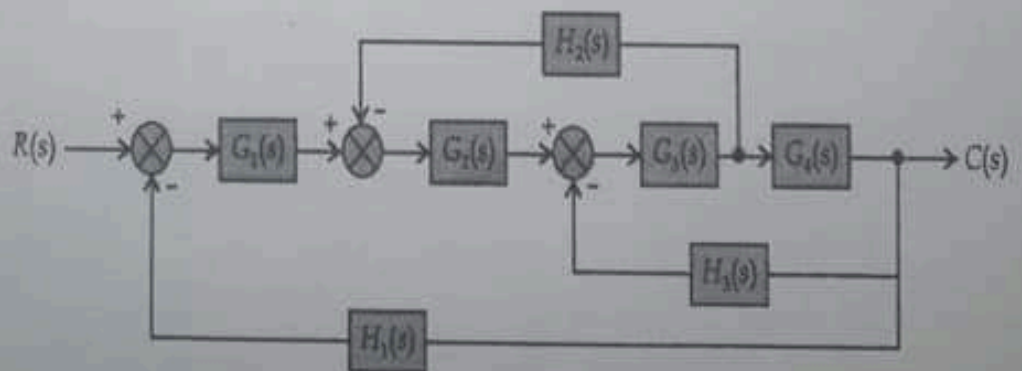


Figure (3)

**Q3 (10 points).** Consider the cascaded liquid storage tanks shown in figure (4). The differential equation describing the behaviour of the first tank is given as  $h_1'(t) = 8(F_{in}(t) - F_{12}(t))$  and the differential equation describing the behaviour of second tank is given as  $h_2'(t) = 10(F_{12}(t) - F_{out}(t))$ . The first tank output flow rate is given as  $F_{12}(t) = 0.0625 h_1(t)$  and the second tank output flow rate is given as  $F_{out}(t) = 0.05 h_2(t)$ . Assuming that the two tanks were empty at initial time, find the following:

- The transfer function that relates  $h_1$  to  $F_{in}$ .
- The transfer function that relates  $h_2$  to  $F_{12}$ .
- The transfer function that relates  $h_2$  to  $F_{in}$ .
- find  $h_2(t)$ , If  $F_{in}(t)$  is a unit step signal.
- The time constant of the first tank, and the time constant of the dynamic between  $h_2(t)$  and  $F_{in}(t)$ .

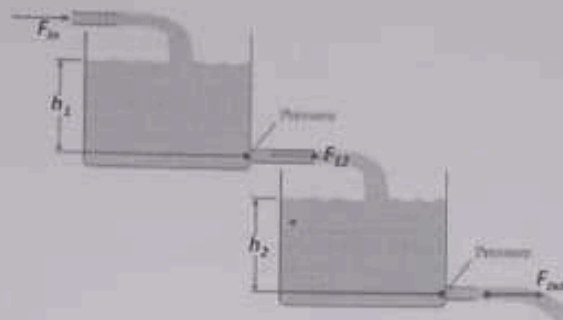


Figure (4)

**Q4 (10 points).** Plot Bode diagram for the open loop system configured in figure (5) with  $G(s) = \frac{250(s+0.04)}{(s+10)(s+1)(s+0.01)}$ . Find also the gain and the phase margins for this system.



Figure (5)

**Q5 (10 points).** For the system of figure (6-a) the forcing function  $F$  is a step input of 2 Newton. Knowing that the transfer function of the system is  $\frac{X(s)}{F(s)} = \frac{1}{m.s^2 + b.s + k}$  where  $m$  is the mass of the body,  $k$  is the spring constant and  $b$  is the friction coefficient. If the response of the system was the function given in figure (6-b) find the values of  $m$ ,  $b$  and  $k$ .

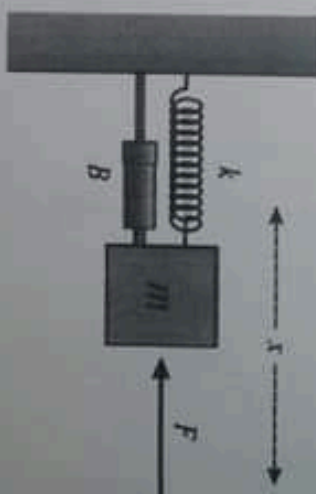


Figure (6-a)

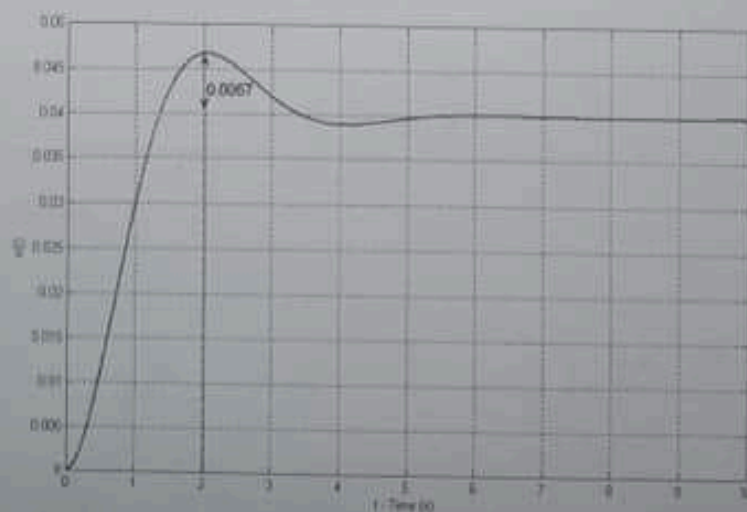


Figure (6-b)

Figure (6)

$\omega$	$\Phi$
0.001	-4.34
0.002	-8.57
0.003	-12.60
0.005	-19.76
0.008	-27.85
0.01	-31.59
0.02	-38.13
0.03	-36.59
0.05	-30.50
0.08	-24.47
0.1	-22.37
0.2	-20.90
0.3	-24.10
0.5	-32.86
0.8	-45.38
1	-52.43
2	-75.60
3	-88.84
5	-105.60
8	-121.75
10	-129.46
20	-150.66
30	-159.71
50	-167.58
80	-172.18
100	-173.73
200	-176.86
300	-177.91
500	-178.74
800	-179.21
1000	-179.37

☑ General form of the first order system:  $G(s) = \frac{Y(s)}{U(s)} = \frac{1}{1 + \tau s}$

☑ General form of the second order system:  $G(s) = \frac{Y(s)}{U(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} = \frac{1}{\tau^2 s^2 + 2\xi\tau s + 1}$

Where  $\xi$  : is the system damping ratio,

$\omega_n = (1/\tau)$  : is the system undamped natural frequency.

☑ The underdamped response case (for  $\xi < 1$ ) of the second order system to a unit step response can be characterized by:

1-Delay time ( $t_d$ ). Is defined as the time required for the response to reach 50% from its ultimate value

2-Rise time ( $t_r$ ). The time required for the response to first reach its ultimate value

$$t_r = \frac{\pi - \beta}{\omega_d} \quad \text{with } \beta = \tan^{-1}\left(\frac{\omega_d}{\xi\omega_n}\right) \quad \text{and } \omega_d = \omega_n\sqrt{1-\xi^2}$$

3-Peak time ( $t_p$ ). The time required for the response to reach its peak  $t_p = \frac{\pi}{\omega_d}$

4-Setting time ( $t_s$ ): If the allowable tolerance is ( 2% )  $\Rightarrow t_s = \frac{4}{\xi\omega_n}$

If the allowable tolerance is ( 5% )  $\Rightarrow t_s = \frac{3}{\xi\omega_n}$

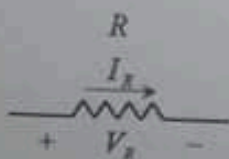
5- Maximum overshoot ( $M_p$ ). Is the measure of how much the response exceeds the ultimate value

following a step input  $M_p = e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}}$

☑ PID controller Parameters (stability limits evaluation.)

Type of controller	Optimum gain
P	$K=0.5K_u$
PI	$K=0.45K_u$ $T_i=P_u/1.2$
PID	$K=0.6K_u$ $T_i=0.5P_u$ $T_D=P_u/8$

☑ Dynamic of basic electrical components :



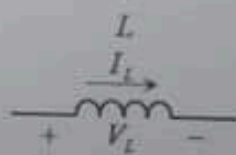
$$V_R(t) = RI_R(t)$$

$$I_R(t) = \frac{V_R(t)}{R}$$



$$V_C(t) = \frac{1}{C} \int_0^t I_C(\tau) d\tau$$

$$I_C(t) = C \frac{dV_C(t)}{dt}$$



$$V_L(t) = L \frac{dI_L(t)}{dt}$$

$$I_L(t) = \frac{1}{L} \int_0^t V_L(\tau) d\tau$$