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| قسم الهندسة الميكانيكية<br>إجابة امتحان نهائي موائع II<br>د. سالم البهلول | كلية الهندسة<br>جامعة مصراتة | العام الجامعي : 19-20<br>التاريخ : 12/12/2020<br>الزمن : 150 دقيقة |

س1 - جريان مستقر ثنائي الأبعاد لمائع نيوتوني غير قابل للانضغاط كثافته ρ ولزوجته ثابتة $\mu$ تعطى سرعته: $\vec{V} = -2xy\vec{i} + (y^2 - x^2)\vec{j}$

أ- اوجد صيغة لمتجه العجلة: $\vec{a} = a_x\vec{i} + a_y\vec{j}$ وأوجد مقداره عند الموضع (1,2)

ب- هل يحقق توزيع السرعة المعطى مبدأ حفظ الكتلة ؟ اثبت ذلك.

ج- أوجد صيغة لتوزيع الضغط $P(x,y)$ علما أن $P(0,0) = P_o$

$$u = -2xy \qquad v = (y^2 - x^2)$$

$$a_x = \frac{du}{dt} = u\frac{du}{dx} + v\frac{du}{dy} = (-2xy)(-2y) + (y^2 - x^2)(-2x)$$

$$a_x = 4xy^2 - 2xy^2 + 2x^3 \rightarrow a_x = 2xy^2 + 2x^3 = 2x(x^2 + y^2)$$

$$a_y = \frac{dv}{dt} = u\frac{dv}{dx} + v\frac{dv}{dy} = -2xy(-2x) + (y^2 - x^2)(2y) \rightarrow a_y = 2x^2y + 2y^3 = 2y(x^2 + y^2)$$

$$a_x(1,2) = 2(1)(1+4) = 10\, units \quad \& \quad a_y(1,2) = 2(2)(1+4) = 20\, units$$

$$a = \sqrt{10^2 + 20^2} = 22.36\, units$$

N.S in x-direction $\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = \rho g_x - \frac{\partial P}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$

$$\frac{\partial u}{\partial x} = -2y \rightarrow \frac{\partial^2 v}{\partial x^2} = 0 \quad \& \quad \frac{\partial u}{\partial y} = -2x \rightarrow \frac{\partial^2 v}{\partial y^2} = 0$$

$$\rho_0[(-2xy)(-2y) + (y^2 - x^2)(-2x)] = \mu(0+0) - \frac{\partial P}{\partial x}$$

$$\frac{\partial P}{\partial x} = -\rho_0(4xy^2 - 2xy^2 + 2x^3) \rightarrow \frac{\partial P}{\partial x} = -\rho_0(2x^3 + 2xy^2)$$

N.S in y-direction $\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) = \rho g_y - \frac{\partial P}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right)$

$$\frac{\partial v}{\partial x} = -2x \qquad \frac{\partial v}{\partial y} = 2y \qquad \frac{\partial^2 v}{\partial x^2} = -2 \qquad \frac{\partial^2 v}{\partial y^2} = 2$$

$$\rho_0[-2xy(-2x) + (y^2 - x^2)(2y)] = \mu(-2+2) - \frac{\partial P}{\partial y}$$

$$\frac{\partial P}{\partial y} = -\rho_0(4x^2y + 2y^3 - 2x^2y) \rightarrow \frac{\partial P}{\partial y} = -\rho_0(2x^2y + 2y^3)$$

Integrating $P = \int \frac{\partial P}{\partial y} dy = \int -\rho_0(2x^2y + 2y^3)\, dy = -\rho_0(x^2y^2 + 0.5y^4) + f(x)$

Differentiating $\frac{\partial P}{\partial x} = -\rho_0(2xy^2) + f'(x) \rightarrow \quad f'(x) = -\rho_0(2x^3)$

$f(x) = -\rho_0(0.5x^4) + c \rightarrow P = -\rho_0(x^2y^2 + 0.5x^4 + 0.5y^4) + c$

Substituting x= 0 & y=0 $P(0,0) = P_o \rightarrow c = P_o \rightarrow$

$$P = -\rho_0(x^2y^2 + 0.5x^4 + 0.5y^4) + P_o$$

س ٢- س2- ينقل زيت ( $\rho = 865\frac{kg}{m^3}$ & $\mu = 1.45\frac{kg}{m.s}$ ) بضخه في خط انابيب نصف قطره 8cm بسرعة 1.2m/s ، إذا كان الجريان كامل النمو (Fully developed) خلال جزء طوله 400m من الأنبوب احسب أقصى سرعة للزيت وكتلة الزيت المنتقلة في الساعة وفواقد الضغط والقدرة اللازمة لاستمرار جريان الزيت في الأنبوب.

$$Re = \frac{\rho V_m D}{\mu} = \frac{1.2 \times 0.16 \times 865}{1.45} = 114.53$$

➔ Laminar flow

$$f = \frac{64}{Re} = \frac{64}{114.53} = 0,559$$

$$V_{max} = 2V_m = 2.4\ m/s$$

$$\dot{V} = V_m A = 1.2\ (\pi\ 0.08^2) = 0.02413\ m^3/s$$

$$\dot{m} = \rho \dot{V} = 865\frac{kg}{m^3} \times 0.02413\ \frac{m^3}{s} \times \frac{3600s}{hr} = 75140\ kg/hr$$

$$\Delta P = f\ \frac{L}{D}\ \frac{\rho V_m{}^2}{2} = 0.559\left(\frac{400}{0.16}\right)\left(\frac{865 \times 1.2^2}{2}\right) = 870.363\ kPa$$

$$\dot{W} = \dot{V}\Delta P = 21kW$$

س ٣ طائرة تجارية كتلتها 70,000kg مساحة سطح الأجنحة $150m^2$ وعند التحليق على ارتفاع ثابت 12km كانت سرعتها 558km/h ، أجنحة الطائرة مزودة بصفين من الرفاف المتحركة (double-slotted flaps) التي يمكن استخدامها أثناء الإقلاع والهبوط لكن لا تستخدم أثناء التحليق (level cruising) بحيث أن متوسط طول الكورد للجناح 3m ومعاملات الكبح والرفع تتغير وفقا للمخططات المرفقة. أوجد

أ- اقصى قيمة لمعامل الرفع للجناح $C_L$ يمكن الحصول عليها أثناء الإقلاع take-off

ب- أقل سرعة يمكن أن تقلع بها الطائرة من الأرض take-off باستخدام معامل أمان 1.2

ج- زاوية ارتطام الهواء بمقدمة الجناح ورقم رينولد أثناء التحليق المستقر عند الارتفاع المذكور

د- القدرة اللازمة للطائرة للتغلب على الكبح الناشئ على الأجنحة أثناء هذا التحليق

خواص الهواء: على سطح الأرض $\left(\rho = 1.2\frac{kg}{m^3} \;\; \& \;\; \mu = 1.8 \times 10^{-5}\frac{kg}{m.s}\right)$

خواص الهواء: على ارتفاع 12km $\left(\rho = 0.312\frac{kg}{m^3} \;\; \& \;\; \mu = 1.21 \times 10^{-5}\frac{kg}{m.s}\right)$

A commercial airplane has a total mass of 70,000 kg and a wing planform area of 150 m$^2$ (Fig. 11–54). The plane has a cruising speed of 558 km/h and a cruising altitude of 12,000 m, where the air density is 0.312 kg/m$^3$. The plane has double-slotted flaps for use during takeoff and landing, but it cruises with all flaps retracted. Assuming the lift and the drag characteristics of the wings can be approximated by NACA 23012 (Fig. 11–45), determine (*a*) the minimum safe speed for takeoff and landing with and without extending the flaps, (*b*) the angle of attack to cruise steadily at the cruising altitude, and (*c*) the power that needs to be supplied to provide enough thrust to overcome wing drag.

***Analysis*** (*a*) The weight and cruising speed of the airplane are

$$W = mg = (70{,}000 \text{ kg})(9.81 \text{ m/s}^2)\left(\frac{1 \text{ N}}{1 \text{ kg·m/s}^2}\right) = 686{,}700 \text{ N}$$

$$V = (558 \text{ km/h})\left(\frac{1 \text{ m/s}}{3.6 \text{ km/h}}\right) = 155 \text{ m/s}$$

The minimum velocities corresponding to the stall conditions without and with flaps, respectively, are obtained from Eq. 11–24,

$$V_{\min 1} = \sqrt{\frac{2W}{\rho C_{L,\max 1} A}} = \sqrt{\frac{2(686{,}700 \text{ N})}{(1.2 \text{ kg/m}^3)(1.52)(150 \text{ m}^2)}\left(\frac{1 \text{ kg·m/s}^2}{1 \text{ N}}\right)} = 70.9 \text{ m/s}$$

$$V_{\min 2} = \sqrt{\frac{2W}{\rho C_{L,\max 2} A}} = \sqrt{\frac{2(686{,}700 \text{ N})}{(1.2 \text{ kg/m}^3)(3.48)(150 \text{ m}^2)}\left(\frac{1 \text{ kg·m/s}^2}{1 \text{ N}}\right)} = 46.8 \text{ m/s}$$

Then the "safe" minimum velocities to avoid the stall region are obtained by multiplying the values above by 1.2:

***Without flaps:*** $V_{\min 1,\text{safe}} = 1.2V_{\min 1} = 1.2(70.9 \text{ m/s}) = 85.1 \text{ m/s} = $ **306 km/h**

***With flaps:*** $V_{\min 2,\text{safe}} = 1.2V_{\min 2} = 1.2(46.8 \text{ m/s}) = 56.2 \text{ m/s} = $ **202 km/h**

since 1 m/s = 3.6 km/h. Note that the use of flaps allows the plane to take off and land at considerably lower velocities, and thus on a shorter runway.

(*b*) When an aircraft is cruising steadily at a constant altitude, the lift must be equal to the weight of the aircraft, $F_L = W$. Then the lift coefficient is

$$C_L = \frac{F_L}{\frac{1}{2}\rho V^2 A} = \frac{686{,}700\text{ N}}{\frac{1}{2}(0.312\text{ kg/m}^3)(155\text{ m/s})^2(150\text{ m}^2)}\left(\frac{1\text{ kg·m/s}^2}{1\text{ N}}\right) = 1.22$$

For the case with no flaps, the angle of attack corresponding to this value of $C_L$ is determined from Fig. 11–45 to be $\alpha \cong 10°$.

(*c*) When the aircraft is cruising steadily at a constant altitude, the net force acting on the aircraft is zero, and thus thrust provided by the engines must be equal to the drag force. The drag coefficient corresponding to the cruising lift coefficient of 1.22 is determined from Fig. 11–45 to be $C_D \cong 0.03$ for the case with no flaps. Then the drag force acting on the wings becomes

$$F_D = C_D A\frac{\rho V^2}{2} = (0.03)(150\text{ m}^2)\frac{(0.312\text{ kg/m}^3)(155\text{ m/s})^2}{2}\left(\frac{1\text{ kN}}{1000\text{ kg·m/s}^2}\right)$$
$$= 16.9\text{ kN}$$

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Noting that power is force times velocity (distance per unit time), the power required to overcome this drag is equal to the thrust times the cruising velocity:

$$\text{Power} = \text{Thrust} \times \text{Velocity} = F_D V = (16.9\text{ kN})(155\text{ m/s})\left(\frac{1\text{ kW}}{1\text{ kN·m/s}}\right)$$
$$= 2620\text{ kW}$$

$\mu$ س4- قوة الكبح لكرة ملساء مغمورة في جريان مائع تعتمد على كل من السرعة النسبية U وقطر الكرة D وكثافة المائع $\rho$ ولزوجته

استخدم التحليل البعدي بنظرية باكنجهام باي لايجاد المعاملات اللابعدية التي يمكن استخدامها لصياغة علاقة تجريبية عمليا . استنتج من ذلك التناسب العكسي بين معامل الكبح ورقم رينولد

**Given:** $F=f(\rho, V, D, \mu)$ for a smooth sphere.

**Find:** An appropriate set of dimensionless groups.

**Solution:** (Circled numbers refer to steps in the procedure for determining dimensionless Π parameters.)

① $F \quad V \quad D \quad \rho \quad \mu$ $\qquad n=5$ dimensional parameters

② Select primary dimensions $M, L$, and $t$.

③ $F \quad V \quad D \quad \rho \quad \mu$

$$\frac{ML}{t^2} \quad \frac{L}{t} \quad L \quad \frac{M}{L^3} \quad \frac{M}{Lt} \qquad r=3 \text{ primary dimensions}$$

④ Select repeating parameters $\rho, V, D$. $\quad m=r=3$ repeating parameters

⑤ Then $n-m=2$ dimensionless groups will result. Setting up dimensional equations, we obtain

$$\Pi_1=\rho^a V^b D^c F \quad \text{and} \quad \left(\frac{M}{L^3}\right)^a\left(\frac{L}{t}\right)^b (L)^c\left(\frac{ML}{t^2}\right)=M^0L^0t^0$$

Equating the exponents of $M, L$, and $t$ results in

$$\left.\begin{aligned} M:&\quad a+1=0 && a=-1\\ L:&\quad -3a+b+c+1=0 && c=-2\\ t:&\quad -b-2=0 && b=-2\end{aligned}\right\}\quad \text{Therefore, } \Pi_1=\frac{F}{\rho V^2D^2}$$

Similarly,

$$\Pi_2=\rho^d V^e D^f \mu \quad \text{and} \quad \left(\frac{M}{L^3}\right)^d\left(\frac{L}{t}\right)^e (L)^f\left(\frac{M}{Lt}\right)=M^0L^0t^0$$

$$\left.\begin{aligned} M:&\quad d+1=0 && d=-1\\ L:&\quad -3d+e+f-1=0 && f=-1\\ t:&\quad -e-1=0 && e=-1\end{aligned}\right\}\quad \text{Therefore, } \Pi_2=\frac{\mu}{\rho VD}$$

⑥ Check using $F, L, t$ dimensions

$$[\Pi_1]=\left[\frac{F}{\rho V^2D^2}\right] \quad \text{and} \quad F\frac{L^4}{Ft^2}\left(\frac{t}{L}\right)^2\frac{1}{L^2}=1$$

where [ ] means "has dimensions of," and

$$[\Pi_2]=\left[\frac{\mu}{\rho VD}\right] \quad \text{and} \quad \frac{Ft}{L^2}\frac{L^4}{Ft^2}\frac{t}{L}\frac{1}{L}=1$$

The functional relationship is $\Pi_1=f(\Pi_2)$, or

$$\frac{F}{\rho V^2D^2}=f\left(\frac{\mu}{\rho VD}\right)$$